Consumption and Labor Supply with Partial Insurance : An Analytical Framework

J Heathcote, K Storesletten and G Violante – AER (2014)

Thomas Bourany

Structural metrics reading group

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Introduction – Motivation

- Quantify the degree of risk-sharing achieved by U.S. Household
- Response of consumption, saving, labor supply to fluctuation in income (insurable or not) and structural

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  2. Insurability nature of the recent increase in US inequality (1967-2006)
     - increase in risk insurability until the 1980s
  3. Life-cycle shocks vs. initial conditions determining inequality
     - Preferences heterogeneity important

- Structural model : artificial laboratory for welfare evaluation
- Consistent theory for conso & hours + Data from PSID and CEX

- Approach : Structural equilibrium model :
  - Different types of shocks : permanent/transitory + insurability
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    - Show a disjuncture between income & consumption inequality in the 1980s ; May be explained by change in persistence of income shocks
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  - Arellano, Blundell, Bonhomme (2017) Earnings and Consumption Dynamics : A Nonlinear Panel Data Framework
    - Generalization in a quantile-based panel study
    - More details about the earning process : non-linear persistence + conditional skewness
    - these drive the heterogeneous responses in consumption

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- Two types of permanent shocks : (partial insurance)
  - Island level shocks : not insurable
  - Individual/idiosyncratic shocks : insurable
- Mechanisms for consumption smoothing :
  - Adjustment in labor supply
  - Borrowing/lending in risk-free bond
  - Government redistribution : progressive taxation
- Provide closed form solution for $c_t(s^t), h_t(s^t), w_t$
- Structural estimation via GMM

Model : Household preferences

- Perpetual youth model, constant survival probability δ
- Continuum of individuals in a continuum of islands
- Preferences over consumption c_t and hours h_t

$$\mathbb{E}_b \sum_{t=b}^{\infty} (\beta\delta)^{t-b} u\left(c_t, h_t; \varphi\right)$$

$$u\left(c_t, h_t; \varphi\right) = \frac{c_t^{1-\gamma} - 1}{1-\gamma} - \exp(\varphi)\frac{h_t^{1+\sigma}}{1+\sigma}$$

- Preference shock : cohort born at time t draws $\varphi_t \sim F_{\varphi t}$ with $\mathbb{V}\text{ar} = \nu_{\varphi t}$

Model : structure of income shocks

- Productivity is composed of an idiosyncratic and island specific components :

$$\log w_t = \underbrace{\alpha_t}_{\text{island}} + \underbrace{\epsilon_t}_{\text{ind.}}$$

- island level follows a random walk (unit root !)

$$\alpha_t = \alpha_{t-1} + \omega_t \quad \text{with} \quad \omega_t \sim F_{\omega t} \qquad \mathbb{V}\text{ar} = \nu_{\omega t}$$

- individual component is formed by a random walk and an i.i.d. transitory

$$\begin{aligned} \epsilon_t &= \kappa_t + \theta_t \quad \text{with} \quad \theta_t \sim F_{\theta t} \qquad \mathbb{V}\text{ar} = \nu_{\theta t} \\ \kappa_t &= \kappa_{t-1} + \eta_t \quad \text{with} \quad \eta_t \sim F_{\eta t} \qquad \mathbb{V}\text{ar} = \nu_{\eta t} \end{aligned}$$

- Productivity : agents entering at time t draw α^0 & κ^0 from cohort specific distributions, with $\mathbb{V}\text{ar} = \nu_{\alpha^0 t}$ and $\nu_{\kappa^0 t}$

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- Between island, limited they can trade, at $t \geq b$, over $s_{t+1} = (\eta_{t+1}, \theta_{t+1})$, but can't condition on ω_{t+1}

- History of shocks : $s^t = s_b, s_{b+1} \dots s_{t+a} \dots s_t \equiv$

$$s_j = \begin{cases} (b, \varphi, \alpha_0, \kappa_0, \theta_b) & \text{for } j = b \\ (\omega_j, \eta_j, \theta_j) & \text{for } j > b \end{cases}$$

Model : Budget and Asset prices

- Gross income : $y_t = w_t h_t$ + CRS production $\Rightarrow$ agents paid MPL w_t
- Yields to net earnings are given by :

$$\widetilde{y}_t = \lambda (y_t)^{1-\tau}$$

- The higher τ, the stronger the redistribution + Approximates well the US tax system

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- Budget :

$$\lambda \left[w_t(\mathbf{s}^t) h_t(\mathbf{s}^t) \right]^{1-\tau} + d_t(\mathbf{s}^t) = c_t(\mathbf{s}^t) + \int Q_t(\mathbf{s}_{t+1};\mathbf{s}^t) B_t(\mathbf{s}_{t+1};\mathbf{s}^t) d\mathbf{s}_{t+1} + \int Q_t^*(\mathbf{z}_{t+1};\mathbf{s}^t) B_t^*(\mathbf{z}_{t+1};\mathbf{s}^t) d\mathbf{z}_{t+1}$$

$$d_t(\mathbf{s}^t) = \delta^{-1} \left[B_{t-1}(\mathbf{s}_t;\mathbf{s}^{t-1}) + B_{t-1}^*(\mathbf{z}_t;\mathbf{s}^{t-1}) \right]$$

- Important assumptions :
  - all assets are in 0 net supply
  - at birth agents have 0 financial asset

Model : Result – Prop 1

- Quite standard definition of equilibrium allocation
- Results :
  (i) Constantinides-Duffie -type of result :
  $\Rightarrow$ no insurance traded between islands $B_t^* (\mathcal{Z}; \mathbf{s}^t) = 0$

Model : Result – Prop 1

- Quite standard definition of equilibrium allocation
- Results :
  (i) Constantinides-Duffie -type of result :
  $\Rightarrow$ no insurance traded between islands $B_t^*(\mathcal{Z}; \mathbf{s}^t) = 0$
  - Where is this no-trade result coming from ?
  - Shocks i.i.d. (common $F_{\omega t}$), multiplicative and unit root (permanent)
  - Power law pref. (but extend to others)
  - Initial wealth degenerated at zero, and zero-net supply.
  - Island dichotomy (either full or no-insurance)
  - Wealth is redundant state variable
  - Fair price for inter-island insurance supports this no-trade (make agents indifferent, c.f. next slide)

- Results - closed form solutions :
  (i) CD no-trade result
  (ii) Consumption and hours are given by closed formulas :

$$\log c_t\left(\mathbf{s}^t\right) = -(1-\tau)\widehat{\varphi} + (1-\tau)\left(\frac{1+\widehat{\sigma}}{\widehat{\sigma}+\gamma}\right)\alpha_t + \widetilde{C}_t^a$$

$$\log h_t\left(\mathbf{s}^t\right) = -\widehat{\varphi} + \left(\frac{1-\gamma}{\widehat{\sigma}+\gamma}\right)\alpha_t + \frac{1}{\widehat{\sigma}}\varepsilon_t + \widetilde{H}_t^a$$

  with age $a = t - b$, age/date-specific constants $\widetilde{C}_t^a$ and $\widetilde{H}_t^a$, tax-weighted Frisch elasticity $\frac{1}{\widehat{\sigma}} \equiv \frac{1-\tau}{\sigma+\tau}$, rescaled pref. $\widehat{\varphi} \equiv \frac{\varphi}{\sigma+\gamma+\tau(1-\gamma)}$

  (iii) Bond/insurance price :

$$Q_t\left(S;s^t\right) = Q_t(S) = \beta\exp\left(-\gamma\Delta C_{t+1}\right)\int_S \exp\left(-\gamma(1-\tau)\frac{1+\widehat{\sigma}}{\widehat{\sigma}+\gamma}\omega_{t+1}\right)dF_{s,t+1}$$

$$Q_t^*\left(Z;s^t\right) = Q_t^*(\mathrm{Z}) = \Pr\left((\eta_{t+1},\theta_{t+1})\in Z\right)\times Q_t(\mathrm{S})$$

  with ΔC_t^a ndependent of a + F_{st} integrates over (ω,η,θ)

Taking the model to the data : Structural estimation

- Modification/Augmentation of the model
  - (i) HH composition/size and (ii) Measurement errors (all terms in μ)
- Data : repeated cross section
  - Moments : time-t and age-a specific, e.g. α

$$\mathbb{V}\text{ar}_t^a(\alpha) = v_{\alpha^0,t-a} + \sum_{j=0}^{a-1} v_{\omega,t-j}$$

  - Use PSID with fine age groups :
    - moments in level involving hours h_t and wages w_t : Macro-moments
    - same moments in difference : dispersion over life cycle : $\Delta\mathbb{V}\text{ar}_t^a(x) = \mathbb{V}\text{ar}_t^a(x) - \mathbb{V}\text{ar}_{t-1}^{a-1}(x)$
    - same moments *of* differences $\mathbb{V}\text{ar}_t^a(\Delta x)$ and second differences $\mathbb{V}\text{ar}_t^a(\Delta^2 x)$
  - Use CEX with fine age groups for
    - moments in level involving consumption c_t (Macro-moments and dispersion over life cycle)

Structural estimation : Macro moments - 1

- Macro moments : (labor)

$$\mathbb{V}\text{ar}_t^a(\log \hat{w}) = \mathbb{V}\text{ar}_t^a(\alpha) + \mathbb{V}\text{ar}_t^a(\varepsilon) + v_{\mu y} + v_{\mu h}$$

$$\mathbb{V}\text{ar}_t^a(\log \hat{h}) = \mathbb{V}\text{ar}_t^a(\widehat{\varphi}) + \left(\frac{1-\gamma}{\widehat{\sigma}+\gamma}\right)^2 \mathbb{V}\text{ar}_t^a(\alpha) + \frac{1}{\widehat{\sigma}^2}\mathbb{V}\text{ar}_t^a(\varepsilon) + v_{\mu h}$$

$$\mathbb{C}\text{ov}_t^a(\log \hat{w}, \log \hat{h}) = \left(\frac{1-\gamma}{\widehat{\sigma}+\gamma}\right) \mathbb{V}\text{ar}_t^a(\alpha) + \frac{1}{\widehat{\sigma}}\mathbb{V}\text{ar}_t^a(\varepsilon) - v_{\mu h}$$

- Macro moments : (consumption)

$$\mathbb{V}\text{ar}_t^a(\log \hat{c}) = (1-\tau)^2\mathbb{V}\text{ar}_t^a(\widehat{\varphi}) + (1-\tau)^2 \left(\frac{1+\widehat{\sigma}}{\widehat{\sigma}+\gamma}\right)^2 \mathbb{V}\text{ar}_t^a(\alpha) + v_{\mu c}$$

$$\mathbb{C}\text{ov}_t^a(\log \hat{h}, \log \hat{c}) = (1-\tau)\mathbb{V}\text{ar}_t^a(\widehat{\varphi}) + \frac{(1-\tau)(1+\widehat{\sigma})(1-\gamma)}{(\widehat{\sigma}+\gamma)^2}\mathbb{V}\text{ar}_t^a(\alpha)$$

$$\mathbb{C}\text{ov}_t^a(\log \hat{w}, \log \hat{c}) = (1-\tau)\left(\frac{1+\widehat{\sigma}}{\widehat{\sigma}+\gamma}\right) \mathbb{V}\text{ar}_t^a(\alpha)$$

Structural estimation : Life-cycle moments - 2

- Life-cycle moments (labor) : get rid of indiv. initial conditions

$$\Delta \mathbb{V}\text{ar}_t^a(\log \hat{w}) = v_{\omega t} + v_{\eta t} + \Delta v_{\theta t}$$

$$\Delta \mathbb{V}\text{ar}_t^a(\log \hat{h}) = \left(\frac{1-\gamma}{\widehat{\sigma}+\gamma}\right)^2 v_{\omega t} + \frac{1}{\widehat{\sigma}^2}\left(v_{\eta t} + \Delta v_{\theta t}\right)$$

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- Life-cycle moments (consumption) :

$$\Delta \mathbb{V}\text{ar}_t^a(\log \hat{c}) = (1-\tau)^2 \left(\frac{1+\widehat{\sigma}}{\widehat{\sigma}+\gamma}\right)^2 v_{\omega t}$$

$$\Delta \mathbb{C}\text{ov}_t^a(\log \hat{h}, \log \hat{c}) = (1-\tau)\frac{(1-\gamma)(1+\widehat{\sigma})}{(\widehat{\sigma}+\gamma)^2} v_{\omega t}$$

$$\Delta \mathbb{C}\text{ov}_t^a(\log \hat{w}, \log \hat{c}) = (1-\tau)\left(\frac{1+\widehat{\sigma}}{\widehat{\sigma}+\gamma}\right) v_{\omega t}$$

Structural estimation : Micro moments - 3

- Micro moment (labor) : link the variance over periods

$$\mathbb{V}\mathrm{ar}_t^a(\Delta \log \hat{w}) = v_{\omega t} + v_{\eta t} + v_{\theta t} + v_{\theta,t-1} + 2v_{\mu y} + 2v_{\mu h}$$

$$\mathbb{V}\mathrm{ar}_t^a(\Delta \log \hat{h}) = \left(\frac{1-\gamma}{\hat{\sigma}+\gamma}\right)^2 v_{\omega t} + \frac{1}{\hat{\sigma}^2}\left(v_{\eta t} + v_{\theta t} + v_{\theta,t-1}\right) + 2v_{\mu h}$$

$$\mathbb{C}\mathrm{ov}_t^a(\Delta \log \hat{w}, \Delta \log \hat{h}) = \left(\frac{1-\gamma}{\widehat{\sigma}+\gamma}\right) v_{\omega t} + \frac{1}{\hat{\sigma}}\left(v_{\eta t} + v_{\theta t} + v_{\theta,t-1}\right) - 2v_{\mu h}$$

$$\mathbb{V}\mathrm{ar}_t^a\left(\Delta^2 \log \hat{w}\right) = v_{\omega t} + v_{\omega,t-1} + v_{\eta t} + v_{\eta,t-1} + v_{\theta t} + v_{\theta,t-2} + 2v_{\mu y} + 2v_{\mu h}$$

Structural estimation - in practice

- Estimate parameters using minimum distance (11, 532 moments and 164 parameters)

$$\min_{\Lambda}[\widehat{m} - m(\Lambda)]'W[\widehat{m} - m(\Lambda)]$$

- Small sample size : $W \equiv Id$ and conf. interval : (block)-bootstrap
- Proposition 2 and corollaries (and proof) : identification
- Experiment where they shut down each channel : show that all the terms matter to match both consumption and labor moments.

Structural estimation - result

Preference Elasticities			Life-Cycle Shocks		
σ	γ		$\overline{v_\omega}$	$\overline{v_\eta}$	$\overline{v_\theta}$
2.165	1.713		0.0056	0.0044	0.043
(0.173)	(0.054)		(0.0008)	(0.0012)	(0.005)
Initial Heterogeneity			**Measurement Error**		
$\overline{v_{\alpha^0}}$	$\overline{v_{\kappa^0}}$	$\overline{v_{\widehat{\varphi}}}$	$v_{\mu y}$	$v_{\mu h}$	$v_{\mu c}$
0.102	0.047	0.054	0.000	0.036	0.041
(0.030)	(0.023)	(0.016)	(0.000)	(0.006)	(0.002)

- Other parameters (external setting) :
  - $\delta = 0.996$, $\tau = 0.185$ ($R^2 = 0.92$), and $1/\widehat{\sigma} = 0.35$ consistent with literature
  - Roughly 45% of permanent shocks appears to be insurable

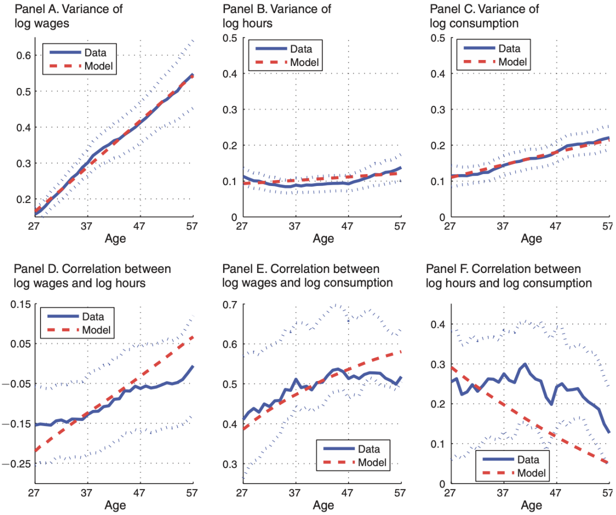

Figure 1. Data and Model Fit for Moments in Levels along the Age Dimension

Structural estimation – Passthrough from income to consumption

- Pass-through from permanent wage shocks to consumption :

$$\underbrace{\phi_t^{w,c}}_{0.386} = \underbrace{\frac{v_{\omega t}}{v_{\omega t} + v_{\eta t}}}_{0.560} \cdot \underbrace{\frac{1 + \hat{\sigma}}{\hat{\sigma} + \gamma}}_{0.845} \cdot \underbrace{(1 - \tau)}_{0.815}.$$

  - progressive taxation 0.815
  - labor supply 0.845
  - private insurance 0.63
- overall $\varphi_t^{w,c} = 0.386$
- the pass-through $\phi_t^{y,c}$ from pre-tax earnings $y = wh$ is 0.272, very similar to Blundell, Pistaferri and Preston which found 0.225

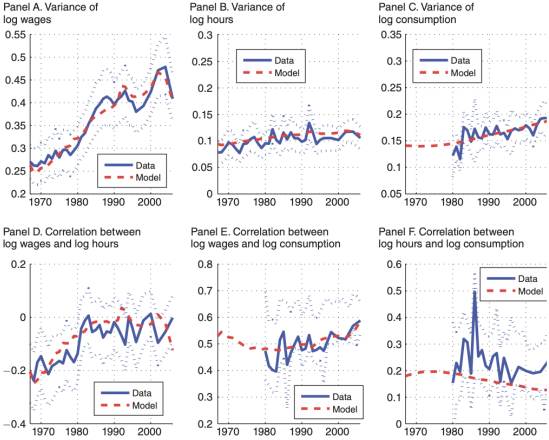

FIGURE 4. DATA AND MODEL FIT FOR MOMENTS IN LEVELS ALONG THE TIME DIMENSION

- Sharp rise in the wage-hours-corr. : rise in variance of insurable wage and fall in variance of uninsurable shocks

Decomposition of inequality

- Initial heterogeneity : explains between 40-50 percent

Table 3—Decomposition of Cross-Sectional Inequality

	Total variance	Percent contribution to total variance					
		Initial heterogeneity			Life-cycle shocks		Measurements
		Preferences	Uninsurable	Insurable	Uninsurable	Insurable	Error
var(log $\hat{w}$)	0.351	0.0	31.5	10.0	17.1	31.3	10.1
var(log $\hat{h}$)	0.107	48.9	2.2	3.2	1.2	9.8	34.7
var(log $\hat{y}$)	0.432	11.7	22.8	12.5	10.4	43.7	0.0
var(log $\hat{c}$)	0.159	20.0	32.6	0.0	17.8	0.0	29.6

Summary

- Tractable framework a la Constantinides and Duffie to study risk-sharing
- Powerful structural GMM estimation
- Match key facts and trends about inequality, income risk and variance in consumption over the life-cycle and over time